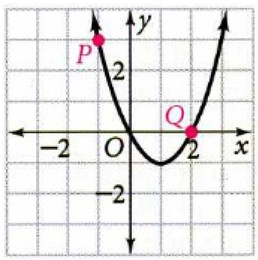
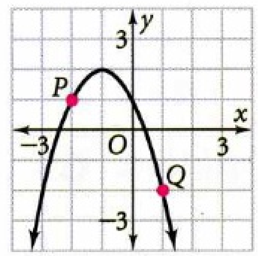
8-1 Exploring Quadratic Graphs

- The graph of a quadratic function $y = ax^2 + bx + c$ is a U-shaped curve called a **parabola**.
- The highest or lowest point of the parabola is called the **vertex**.

Example 1: Identify the vertex of each graph. Tell whether it is a maximum or a minimum.

a) 

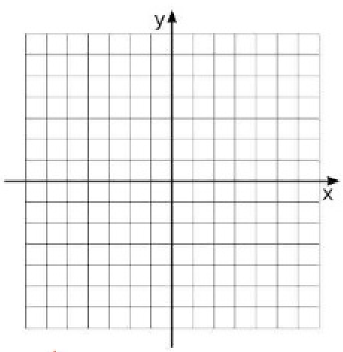
b) 

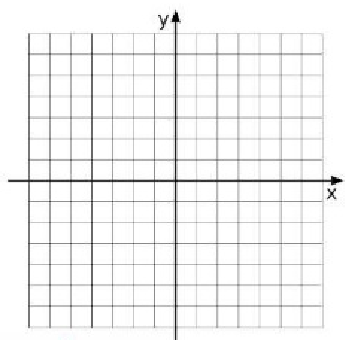
Example 2: Make a table of values and graph each function. Find the vertex. Is the vertex a maximum or a minimum? Can you tell (without graphing) if your vertex is going to be a maximum or a minimum?

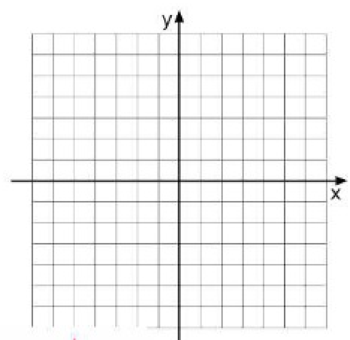
a) $y = x^2$

b) $y = \frac{1}{2}x^2$

c) $y = -2x^2$







x	y
-2	
-1	
0	
1	
2	

x	y
-2	
-1	
0	
1	
2	

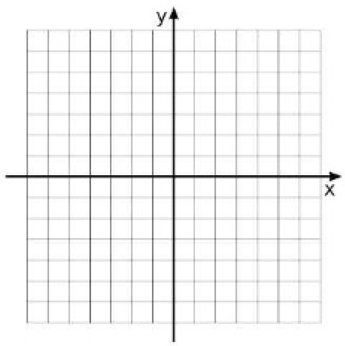
x	y
-2	
-1	
0	
1	
2	

Example 3: Without graphing, order the quadratic functions from widest to narrowest:

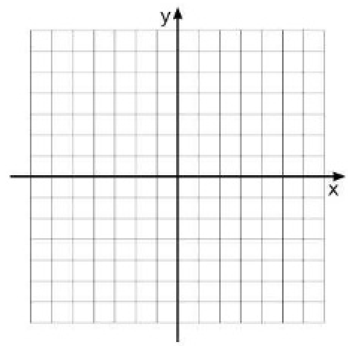
$$y = -4x^2, \quad y = \frac{1}{4}x^2, \quad y = x^2$$

Example 4: Graph the following functions. Compare the graphs.

a) $y = x^2$

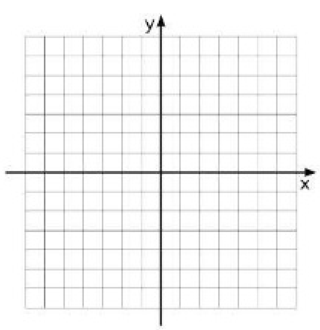


b) $y = x^2 - 4$

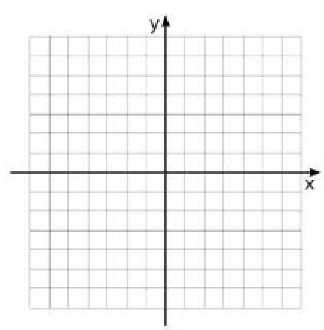


Example 5: Graph the following functions. Compare the graphs.

a) $y = 2x^2$



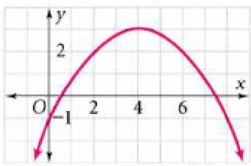
b) $y = 2x^2 + 3$

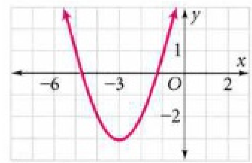


8-2 Quadratic Functions (Part # 1)

- The **vertex** is the highest or lowest point on the graph.
- The **axis of symmetry** is the vertical line that splits the parabola down the middle.

Example 1: Find the vertex and the axis of symmetry for the following graphs.

a) 

b) 

Vertex Formula: The graph of $y = ax^2 + bx + c$ has the line $x = -\frac{b}{2a}$ as its axis of symmetry. The x-coordinate of the vertex is $x = -\frac{b}{2a}$. You can find the y by plugging x into your equation.

Example 2: Find the vertex and the axis of symmetry for the following functions.

a) $y = 2x^2 + 4x$ b) $y = -x^2 + 4x - 5$

Up/ Down Test The graph of $y = ax^2 + bx + c$ opens upwards if a is ________________ and opens downward if a is ________________.

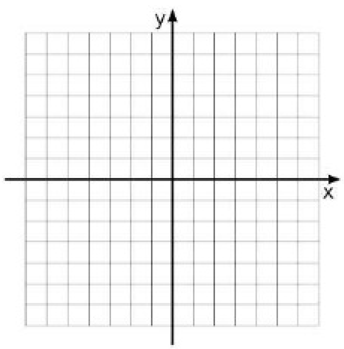
Example 3: Determine whether the following functions open upward or downward.

a) $y = x^2 + 3x + 4$ b) $y = -3x^2 + 5x$ c) $y = 2x - x^2 + 6$

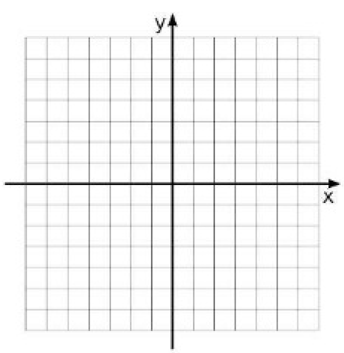
Steps to Graph $y = ax^2 + bx + c$

- Find the vertex and the axis of symmetry. Sketch these in.
- Find the x-intercept by plugging in 0 for y.
- Find the y-intercept by plugging in 0 for x.
- Reflect your points across the axis of symmetry and connect your dots with a smooth U-shaped (not V-shaped) curve.

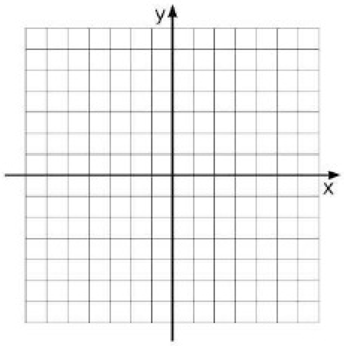
Example 4: Graph $f(x) = x^2 - 2x - 8$



Example 5: Graph $y = -x^2 + 2x + 3$



Example 6: Graph $y = 2x^2 - 8x$



Example 7: Suppose a particular "star" is projected from a firework at a starting height of 520 feet with an initial upward velocity of 72 ft/sec. The equation

$$h = -16t^2 + 72t + 520$$

gives the star's height h in feet at time t in seconds.

a) How long will it take for the star to reach its maximum height?

b) What is the maximum height?

Practice:

8-2 Quadratic Functions (Part # 2)

- The axis of symmetry is the vertical line $x = -\frac{b}{2a}$. The axis of symmetry divides the parabola in two equal halves.
- The vertex is the point (x, y) where $x = -\frac{b}{2a}$. We then use this $x-$value in the equation to find $y-$value of the vertex. The vertex is the highest or lowest point on the curve.

Example 1: Find the equation of the axis of symmetry and the coordinates of the vertex. Does the parabola open up or down? Is the vertex a minimum or a maximum?

a) $y = x^2 + 14x - 9$

b) $y = -4x^2 + 24x + 6$

Example 2: Find the equation of the axis of symmetry and the coordinates of the vertex. Does the parabola open up or down? Is the vertex a minimum or a maximum?

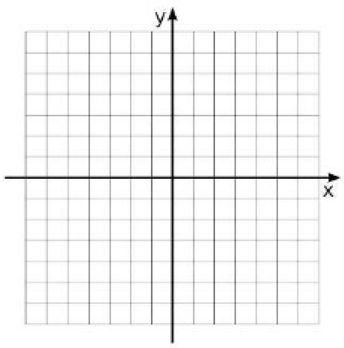
a) $y = 16x - 2x^2$

b) $y = 5x^2 - 3$

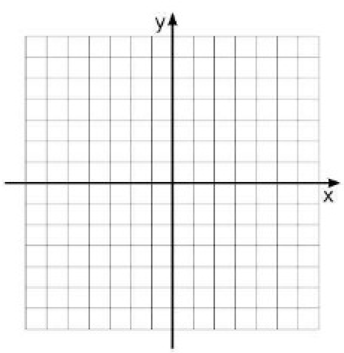
Steps for Graphing $y = ax^2 + bx + c$

1. Find the vertex and axis of symmetry. You use ____________ to find x and to find y you ____________.
2. Find the x-intercepts. Do this by plugging in ____________.
3. Find the y-intercepts. Do this by plugging in ____________.
4. Reflect any points, connect the dots.

Example 3: Graph $y = x^2 - 6x + 5$

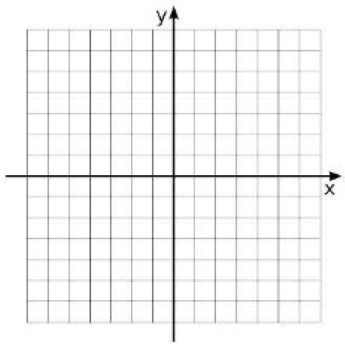


Example 4: Graph $y = -x^2 + 4x - 3$

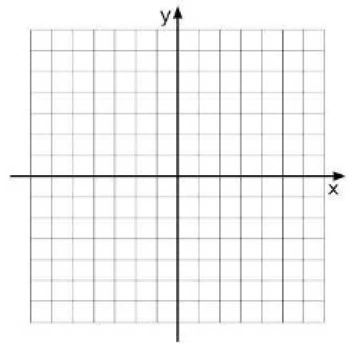


Example 5: Graph the following quadratic functions.

a) $y = 4x^2 + 8x$



b) $y = -2x^2 + 3$



Example 6: The total profit made by an engineering firm is given by the equation

$$p = -x^2 + 24x + 5000$$

where x is the number of clients the firm has and p is the profit. Find the maximum profit made by the company.

Practice: 8-2 Quadratic Functions Worksheet

Practice: 8-2 Quadratic Functions Worksheet

Find the equation of the axis of symmetry and the coordinates of the vertex.

1. $y = x^2 - 10x + 2$
2. $y = x^2 + 12x - 9$
3. $y = -x^2 + 2x + 1$
4. $y = 3x^2 + 3$
5. $y = 16x - 4x^2$
6. $y = 0.5x^2 + 4x - 2$
7. $y = -1.5x^2 + 6x$

Graph each function. Label the axis of symmetry and the vertex.

8. $y = x^2 - 6x + 5$
9. $y = x^2 + 4x + 3$
10. $y = -x^2 - 4x - 4$
11. $y = x^2 - 2x - 8$
12. $y = 4x^2 + 8x$
13. $y = 2x^2 + 4$
14. You and a friend are hiking in the mountains. You want to climb a ledge that is 20 feet high. The height of the grappling hook you throw is given by the function

$$h = -16t^2 + 32t + 5.$$

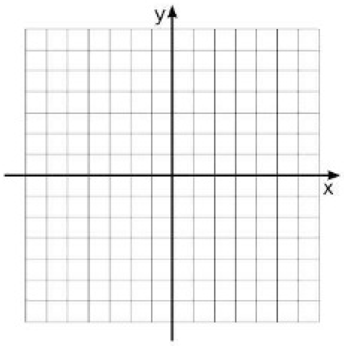
What is the maximum height of the grappling hook? Can you throw it high enough to reach the ledge?

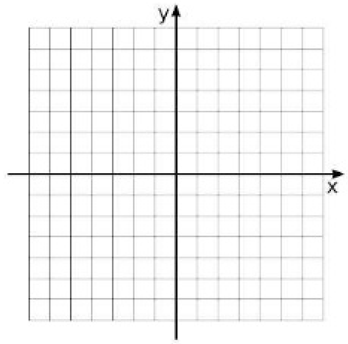
15. You are trying to dunk a basketball. You need to jump 2.5 feet in the air to dunk the ball. The height of your feet above the ground is given by the function

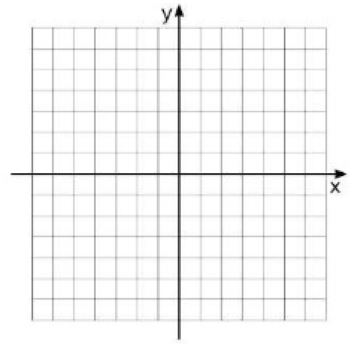
$$h = -16t^2 + 12t.$$

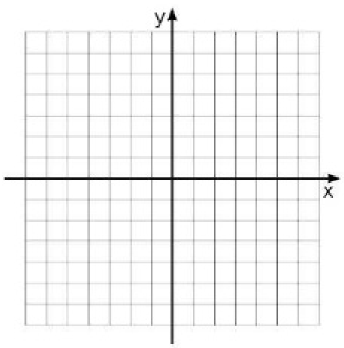
What is the maximum height of your feet above the ground? Will you be able to dunk the basketball?

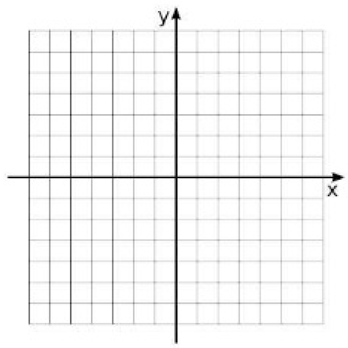
Name:

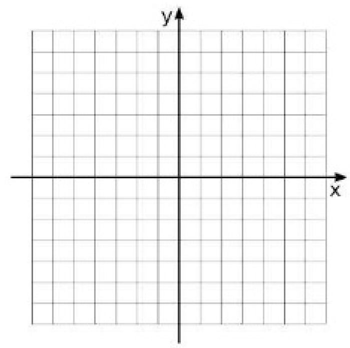


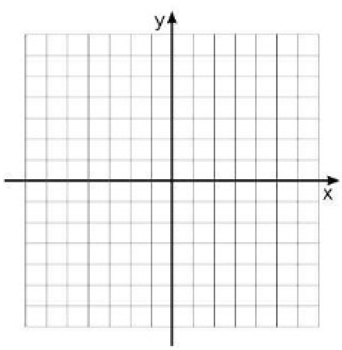


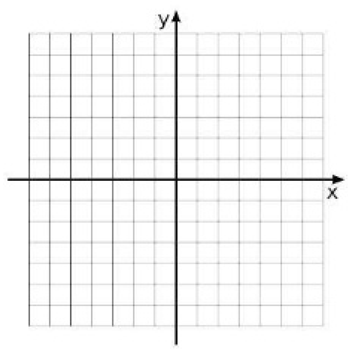


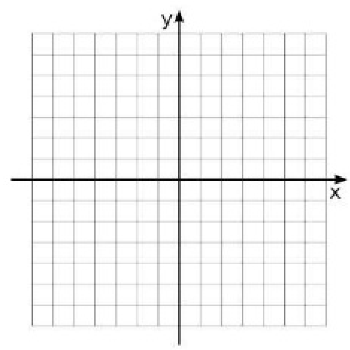


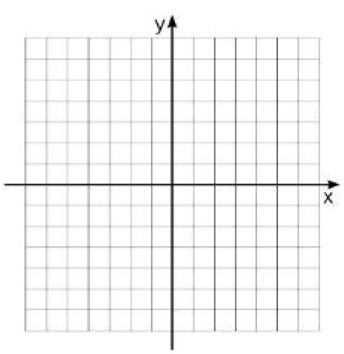


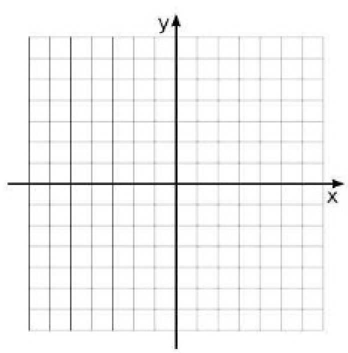


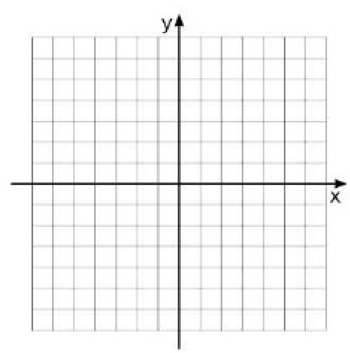








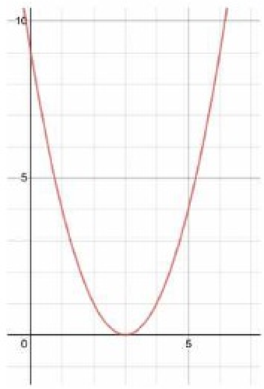




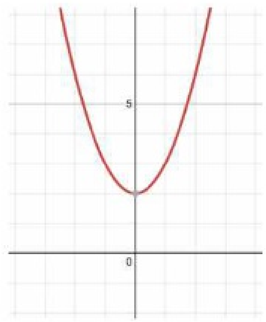
8-3 Finding $x-$Intercepts of Quadratic Functions (Part # 1)

- The $x-$**intercepts** of a parabola are the points where the graph intersects with the $x-$axis. Equivalently, the $x-$intercepts are the points on the graph where $y = 0$.
- A parabola can have ________, ________, or ________ $x-$intercepts.

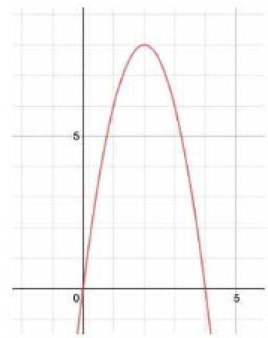
Example 1: Identify and label the $x-$intercepts of each graph.



a)



b)



c)

Example 2: Suppose that you multiply two numbers and the result is zero. What can we say for sure about the numbers you multiplied?

Zero Product Property If the product of two (or more) numbers is equal to zero, then one of the numbers must be zero.

Example 3: We can use the Zero Product Property to find the $x-$intercepts of the graph of a polynomial function. We do this by substituting $y = 0$ and factoring the expression! Find the $x-$intercepts of each parabola.

a) $y = 2x^2 + 4x$

b) $y = x^2 - 4x + 5$

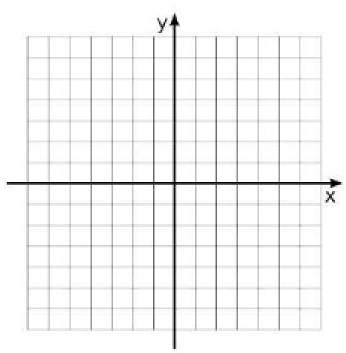
Steps to find $x-$intercepts of factorable quadratic functions:

- Write the equation of the function in standard form: $y = ax^2 + bx + c$
- Substitute $y = 0$.
- Factor the expression $ax^2 + bx + c$.
- Set the resulting factors equal to zero and solve for x.

Example 3: Find the $x-$intercepts of each function.

a) $y = x^2 + 4x + 4$ b) $y = -3x^2 + 6x$ c) $y = 2x^2 + x - 6$

Example 4: Graph $f(x) = x^2 - 2x - 8$



Example 5: Suppose model rocket is launched from a platform 128 feet off the ground with an initial upward velocity of 32ft/sec. The equation $h = -16t^2 + 32t + 128$ gives the rocket's height h in feet at time t in seconds. When will the rocket hit the ground?

Practice: Finding $x-$Intercepts Worksheet

Practice: 8-3 Finding $x-$Intercepts Worksheet #1

Find the $x-$intercepts of each parabola.

1. $y = x^2 - 6x + 9$
2. $y = x^2 + x - 9$
3. $y = -x^2 + 2x - 1$
4. $y = 3x^2 - 3$
5. $y = 16x - 4x^2$
6. $y = 4x^2 + 11x + 6$
7. $y = x^2 + 6x$

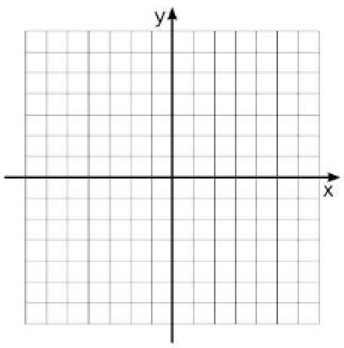
Graph each function. Label the axis of symmetry, the $x-$intercepts, and the vertex.

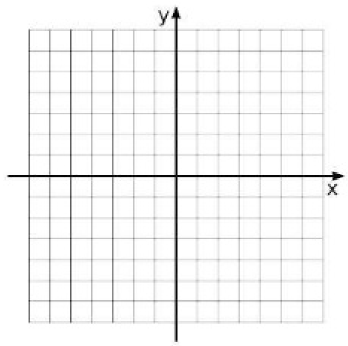
8. $y = x^2 - 6x + 5$
9. $y = x^2 + 4x + 3$
10. $y = -x^2 - 4x - 4$
11. $y = x^2 - 2x - 8$
12. $y = 4x^2 + 8x$
13. $y = x^2 - 4$
14. You and a friend are hiking in the mountains. You want to climb a ledge that is 20 feet high. The height of the grappling hook you throw is given by the function

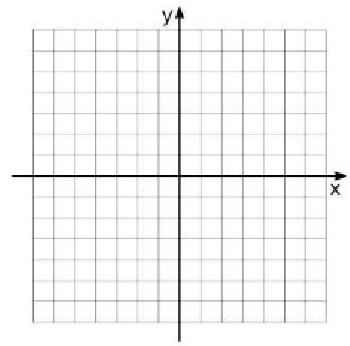
$$h = -16t^2 + 38t + 5.$$

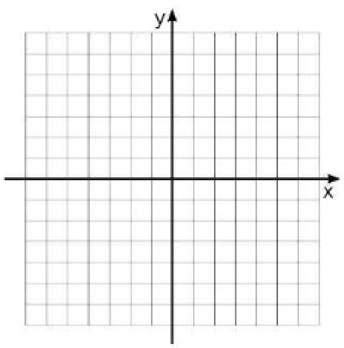
We already know you can throw it high enough, but what if you miss? After how many seconds will the hook land back where you are standing?

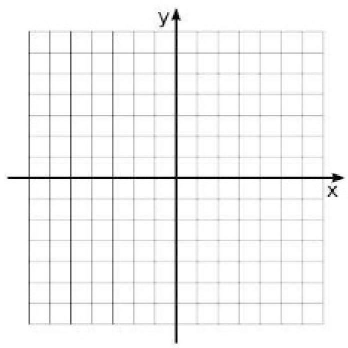
Name:

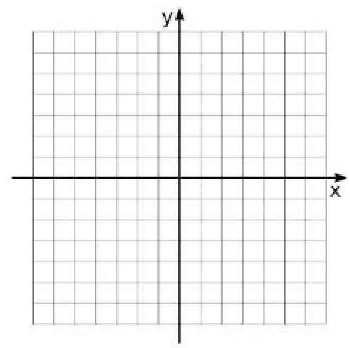


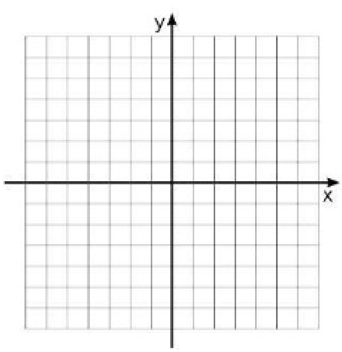


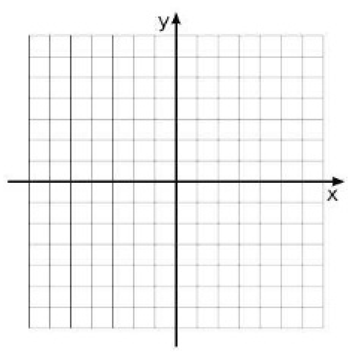


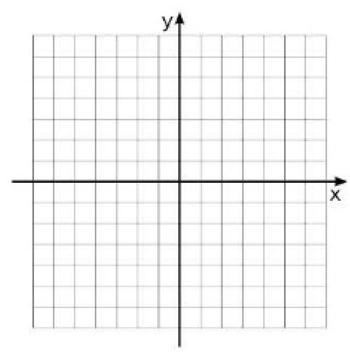


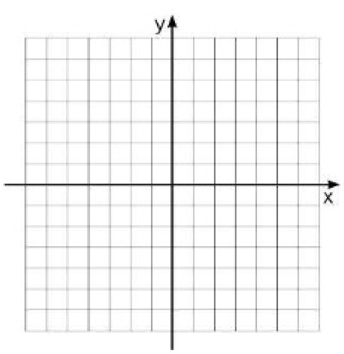


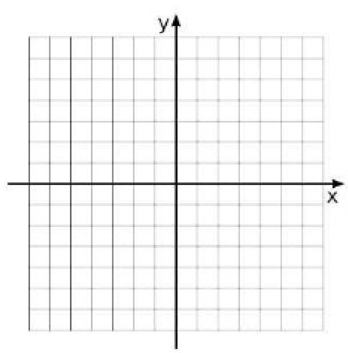


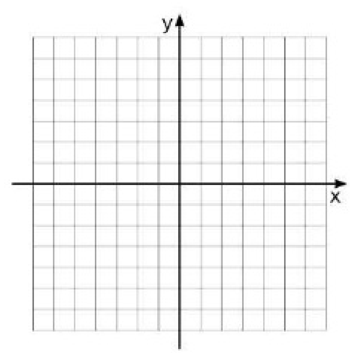








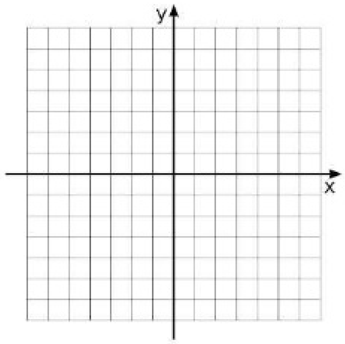




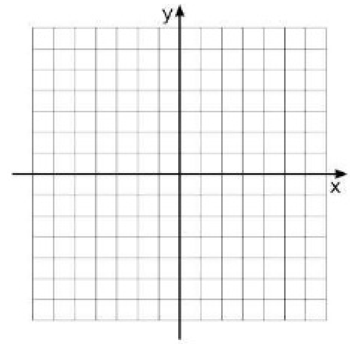
8-4 Vertex Form and Transformations (Part 1)

Example 0: Graph the functions. Recall that $x = \frac{-b}{2a}$ gives the $x-$coordinate of the vertex.

a) $y = x^2 - 6x + 8$



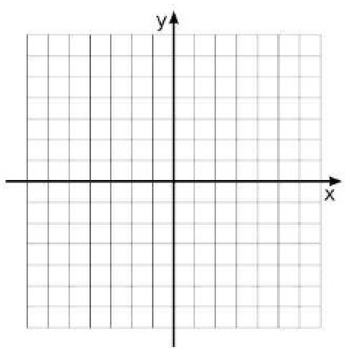
b) $f(x) = -2x^2 - 4x - 2$



Example 1: Make a table to graph the following functions.

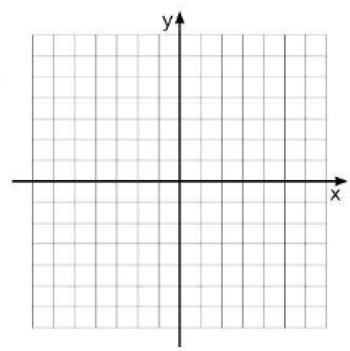
a) $y = (x-3)^2 - 1$

x	y
1	
2	
3	
4	
5	



b) $f(x) = -2(x+1)^2$

x	y
-3	
-2	
-1	
0	
1	



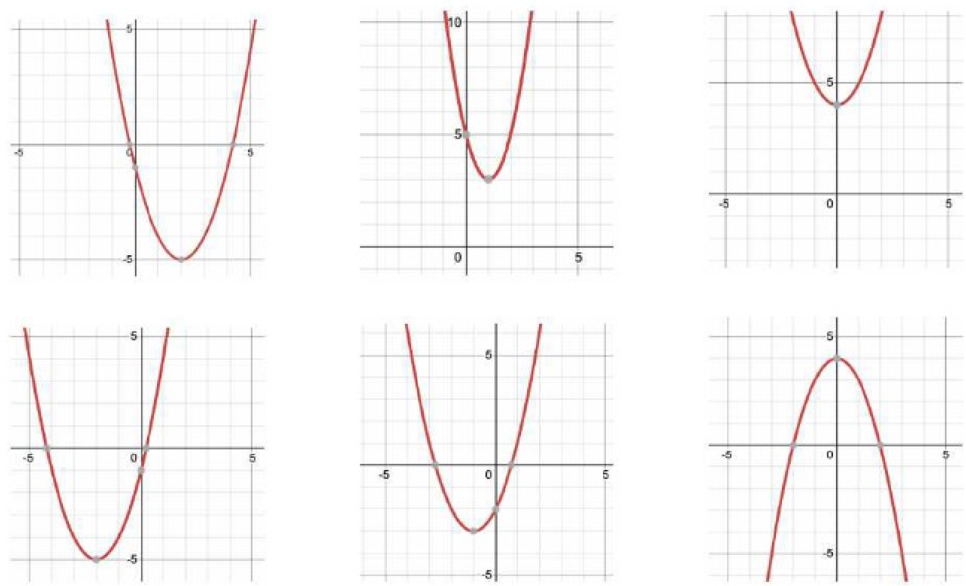
Vertex Form: The *vertex form* of a quadratic function is given by

$$f(x) = a(x-h)^2 + k$$

where (h, k) is the vertex of the parabola and a describes the orientation and stretch or compression compared to the graph of $y = x^2$.

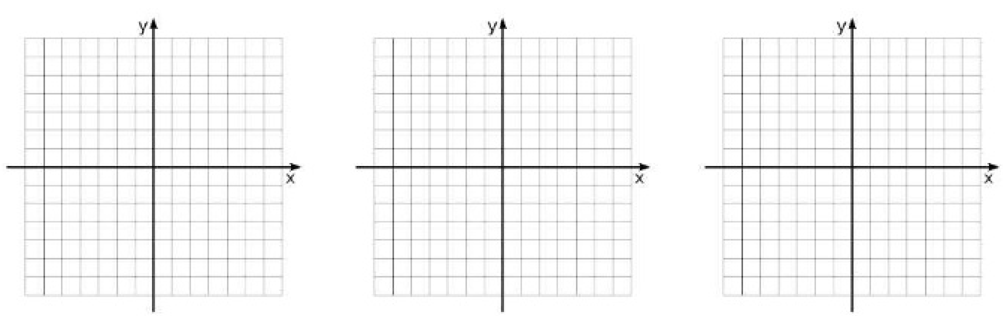
Example 2: Identify the vertex of each parabola from the equation. Then match each equation with its graph.

a) $f(x) = (x+2)^2 - 5$ b) $g(x) = 2(x-1)^2 + 3$ c) $h(x) = -x^2 + 4$



Example 3: Sketch the graph of each parabola. Show at least 5 precise points.

a) $y = x^2$ b) $f(x) = (x+3)^2$ c) $f(x) = (x-4)^2 - 1$

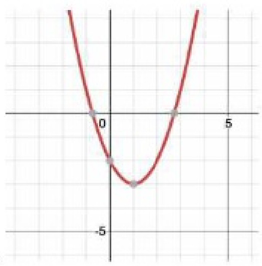


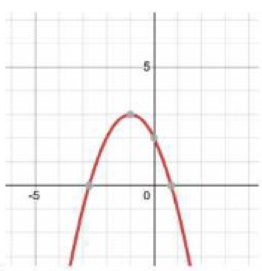
Practice: 8-4 Vertex Form Worksheet # 1

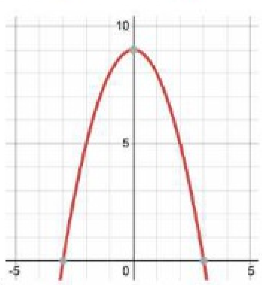
8-4 Vertex Form Worksheet # 1

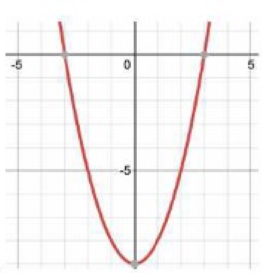
Identify the vertex of each parabola from the equation. Then match each equation with its graph.

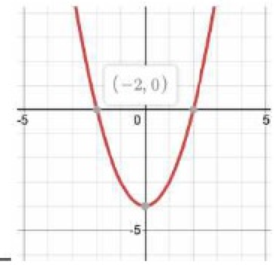
1. $f(x) = x^2 - 4$
2. $y = 3(x+2)^2 + 1$
3. $g(x) = -x^2 + 9$
4. $h(x) = (x-3)^2 - 2$
5. $y = (x-1)^2 - 3$

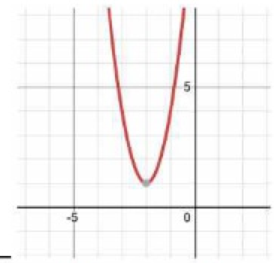
- ___ 

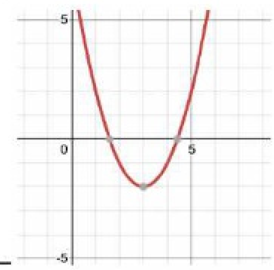
- ___ 

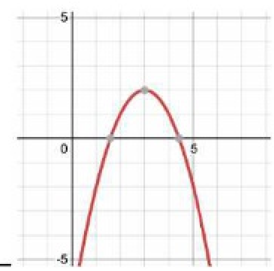
- ___ 

- ___ 

- ___ 

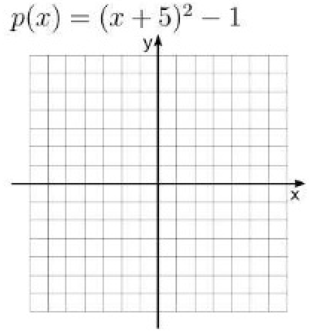
- ___ 

- ___ 

- ___ 

6. Sketch the graph of the function. Show at least 5 precise points on the parabola.

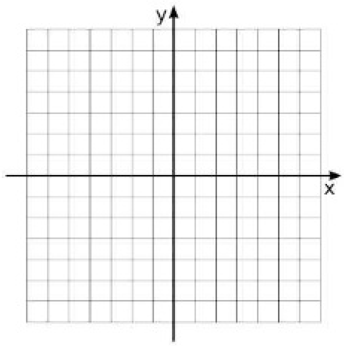
$p(x) = (x+5)^2 - 1$



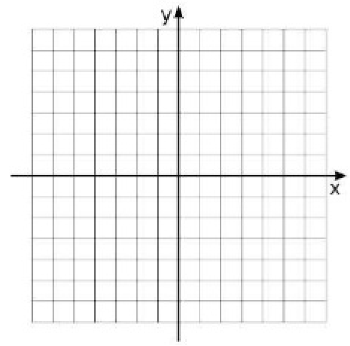
8-4 Vertex Form and Transformations (Part 2)

Example 1: Graph the following functions. First identify the vertex, then find points nearby. Include at least 5 precise points on the parabola.

a) $y = (x + 4)^2 - 2$



b) $f(x) = 2x^2 - 9$



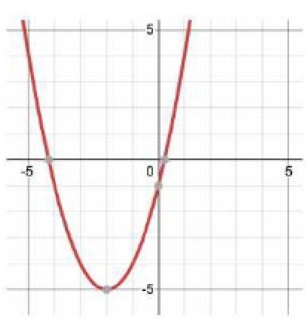
Vertex Form: The *vertex form* of a quadratic function is given by

$$f(x) = a(x - h)^2 + k$$

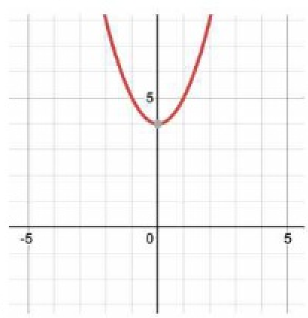
where _____ is the vertex of the parabola and a describes the orientation and stretch or compression compared to the graph of $y = x^2$.

Example 2: Write the equation for each parabola in vertex form.

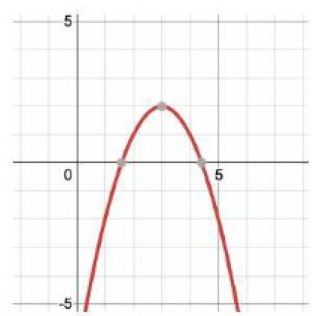
a)



b)



c)



Reflection, Compression, and Stretch: Given $f(x) = a(x-h)^2 + k$,

- If $a > 0$, the parabola ____________________.
- If $a < 0$, the parabola ____________________.
- If $|a| > 1$, the parabola ____________________ compared to the graph of $y = x^2$.
- If $|a| < 1$, the parabola ____________________ compared to the graph of $y = x^2$.

Example 3: Describe the transformations needed to obtain $g(x)$ from the graph of $y = x^2$.

a) $g(x) = 2x^2 - 3$

c) $g(x) = -\frac{1}{2}x^2$

e) $g(x) = (x+1)^2 - 3$

b) $g(x) = 2(x-3)^2 - 1$

d) $g(x) = (x+5)^2$

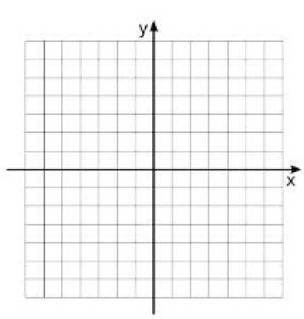
f) $g(x) = -2(x-4)^2 - 7$

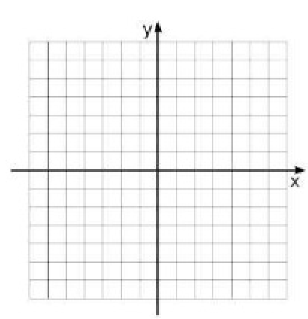
Example 4: Sketch the graph of each parabola using transformations. Show at least 5 precise points.

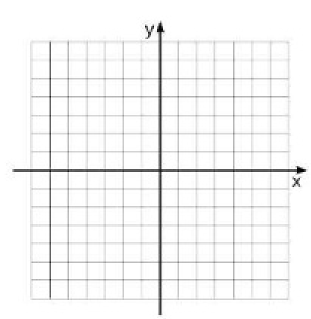
a) $y = -x^2 + 4$

b) $f(x) = 2(x+3)^2$

c) $f(x) = \frac{1}{2}(x-4)^2 - 1$



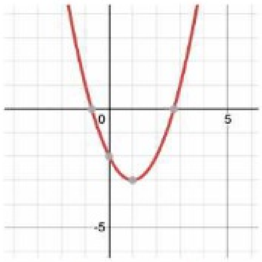


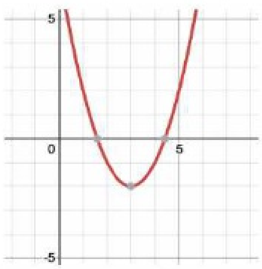


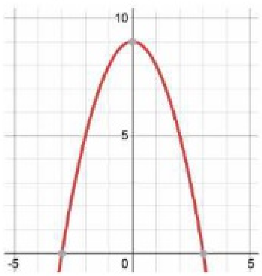
Practice: 8-4 Vertex Form Worksheet # 2

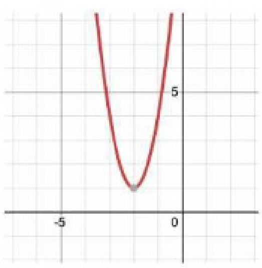
8-4 Vertex Form Worksheet # 2

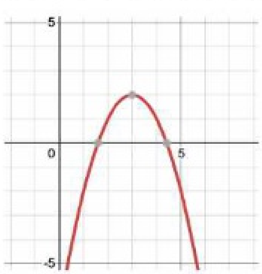
Write the equation for each parabola in vertex form.

1. 

2. 

3. 

4. 

5. 

Describe the transformations needed to obtain $g(x)$ from the graph of $y = x^2$. *It is fine to use the grapher at desmos.com to check!*

6. $g(x) = x^2 + 9$

7. $g(x) = 3(x+2)^2 + 1$

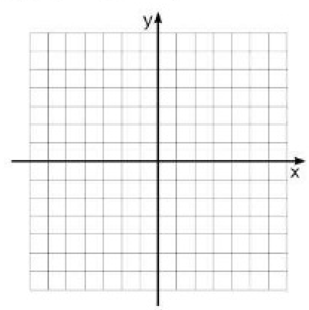
8. $g(x) = -x^2 + 1$

9. $g(x) = -2(x-3)^2 - 2$

10. $g(x) = \frac{1}{2}(x-1)^2 - 3$

Sketch the graph of each parabola using transformations. Show at least 5 precise points.

11. $f(x) = (x+5)^2 - 1$



12. $h(x) = 2(x-3)^2 - 4$